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Beneficiary: Colegiul Național "Constantin Diaconovici Loga", Timișoara

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Maths in Physics

Contents

Foreword.....	2
Introduction	4
Theoretical background	7
Linear Equation or first-degree equation in one variable.	7
The second degree equation	7
Geometric Shapes - Areas.....	14
Operations on vectors	18
Theoretical background in physics.....	20
Velocity–time graphs	21
Circular motion.....	22
Satellites	23
Throwing an object obliquely	24
1. Projectile Motion with an Initial Horizontal Velocity	31
2. Speed and Distance	32
3. MARCO the DREAMer	33
4. Angry Birds Projectile Motion.....	34
5.Movement of artificial satellites around Earth	35
The team	1

Foreword



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This intellectual output was created in the Erasmus project " DREAM - Discover Real Everywhere Applications of Maths", identification number: 2016-1-RO01-KA201-024518, through the collaboration of students and teachers from Colegiul Național "Constantin Diaconovici Loga" Timișoara, Romania, Io Geniko Lykeio, Aigiou, Greece, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis Norte, Portugal and "TIBISCUS" University of Timișoara, Computers and Applied Computer Science Faculty.

The project's main objective was to build up a new Maths teaching/learning methodology based on real-life problems and investigations (open-ended Maths situations), designed by students and teachers together. The activities involved experiments, hands-on approach, outdoor activities and virtual and mobile software applications. The developed material was transformed into online courses and is freely available to all interested communities, in order to produce collaborative learning activities.

O6 - Maths in Physics has the purpose to facilitate the understanding of the usefulness of some mathematical chapters that are applicable to Physics and real life.

The activities in this pack feed into the Skills and Capability Framework by providing contexts for the development of Thinking, Problem Solving and Decision Making Skills and Managing Information. Open-ended questions facilitate pupils to use Mathematics. ICT opportunities are provided through using Moodle platform and additional tasks researching information using the internet.

This intellectual output comprises five lesson scenarios and guides the teacher in creating interactive and exciting lessons.



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Introduction

Can Mathematics provide the answers for Physics? Obviously, yes, because Physics uses makes use of Maths all the time. But is Mathematics enough? Can it give all the answers Physics demands of it? This is also arguably true, because no Physical truth can be thoroughly perceived outside of Mathematics.

So Physics is an important chapter of Mathematics. The other parts of Maths (different from Physics) is an interesting theoretical artifice whose link with Physics has not yet been discovered. Therefore... this other part of Maths doesn't yet have an equivalent in the real world, since if it did, it would also be uncaptured by a truth in Physics...

They start from diametrically opposed places in their quest for the truth: Maths from the abstract and Physics- from the concrete. The relationship between Mathematics and Physics is, therefore, the same as that between theory and practice.

Physics can be taught in such a way that the teacher becomes a revealer of knowledge and truth and a partner of study for the student. More precisely, the lesson may start with the teacher launching a provocative question to the students ('cognitive conflict'). Throughout the lesson, the students would be encouraged to discover the answer by themselves. Their main task would be to find the explanation for the physical phenomenon that was discussed.

Teachers also have the role to activate the a-priori, naive and prescientific knowledge/intuition of children. This working method contributes to a better understanding of the studied phenomena, being used in everyday life.

Benefits: The teaching method described above reawakens not only the students' interest for natural sciences, but also their curiosity about the world around them. Moreover, the majority of students taught in this way would be able to recognize the practical relevance of what they are studying in the exact sciences.

The proposed activities also contribute to important elements used in the current economical environment: critical thinking, creativity, initiative, solving unstandardized problems, and risk evaluation, decision-making. Memory and the use of simple, repetitive procedures have gained a secondary



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role in a lot of jobs. People may now change their career even 3 or 4 times throughout their active lives.

The learning process has been met with new dimensions, such as the developing industry of e-learning. Consequently, platforms with systems managing the learning process have become extremely numerous, so there are multiple options to choose from for anyone interested.

Moodle is one of the most popular of such platforms. It is an open-source, being permanently modified and improved. (Its name stands for Modular Object-Oriented Dynamic Learning Environment.) It can be downloaded for free, its licence being under GNU's General Public License. However, in order to benefit from certain options, an extra tax may be paid.

A big advantage of *Moodle* is its availability in a wide variety of languages. It allows teachers to test their students through homework and written exams. The main difference between it and other similar services is that, besides the pre-set choices or the short answers demanded; students can also upload larger files, such as documents, sheets, images, audio or video.

Teachers have the possibility to leave a comment about the respective homework or organize discussion groups focusing on the subject. Therefore, this platform sets itself apart with the high level of interaction it offers.

With the current project, DREAM, we intend to encourage quality and efficiency in computer and technology-assisted learning, offering theoretical support and spreading these experiences to our partner schools.

The activities in this pack feed into the Skills and Capability Framework by providing contexts for the development of Thinking, Problem Solving and Decision Making Skills and Managing Information. Open ended questions facilitate pupils' Using Mathematics. ICT opportunities are provided through using the Moodle platform and additional tasks researching information using the internet.

Opportunities exist to develop the Key Elements of:

- Employability – exploring how the skills developed in Mathematics will be useful for business records; demonstrating how to be enterprising when discussing potential fund raising activities.
- Curiosity- making young pupils discover answers to the mysteries surrounding them by themselves



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- Model development- once a real-world problem is identified, its solution may be obtained by creating an efficient/creative mathematical model
- Practicability – making students understand the relevance of what they are studying for the real world

Learning and teaching Mathematics can be made more effective when a balance of practical, oral and written tasks is provided. This pack provides information and scenarios to assist in this task. The intention is to provide young people with 8 activities that are related to their age and attainment. One aspect of the pack is the use of the PowerPoint presentations in order to stimulate whole-class discussions before and after the activities have been completed. The emphasis should be on helping young people understand what the problems are and to become aware of the technical vocabulary surrounding the issues.

General Pedagogical Recommendations:

- Watching a PowerPoint presentation or a film, which introduces the theme of real-life lesson
- Discovering the link between real life and the mathematical concept that governs the given situation
- Recall theoretical mathematical concepts
- Frontal discussion of the real situation in the matter
- Solving some parts of the problem by group of students using mathematical tools: minicomputers, Geogebra, Excel, internet
- Discussing solutions, looking for the optimal option
- Student's task: loads the optimal solution found on the MOODLE platform
- Teacher's task: controls the homework of the student and provides feedback.

<http://www.elearning.ro/platforma-elearning-moodle>

<https://ceae.ro/proiect-invatarea-fizicii/>



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Theoretical background

Linear Equation or first-degree equation in one variable.

Definition. A linear or first-degree equation (LE) in one variable x is an equation that can be written in the standard form

$$ax + b = 0, \text{ where } a \text{ and } b \text{ are real numbers and } a \neq 0$$

Linear equations are also called **first degree equations**, as the highest power of the variable (or pronumeral) in these equations is 1.

E.g. $x + 5 = 9$ is an equation of the first degree, which is often called a linear equation.

Many problems can be solved by using linear equations.

Recall that:

Equations behave like a balance. So we need to apply the same **operation** to both sides of an equation to maintain the balance. This means we can:

- add the same number to both sides of an equation
- subtract the same number from both sides of an equation
- multiply both sides of an equation by the same number
- divide both sides of an equation by the same number

The second degree equation

Often, the simplest way to solve " $ax^2 + bx + c = 0$ " for the value of x is to factor the quadratic, set each factor equal to zero, and then solve each factor. But sometimes the quadratic is too messy, or it doesn't factor at all, or you just don't feel like factoring. While factoring may not always be successful, the Quadratic Formula can always find the solution.

The Quadratic Formula uses the " a ", " b ", and " c " from " $ax^2 + bx + c$ ", where " a ", " b ", and " c " are just numbers; they are the "numerical coefficients" of the quadratic equation they've given you to solve.

The Quadratic Formula: For $ax^2 + bx + c = 0$, the values of x which are the solutions of the equation are given by:



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the Quadratic Formula to work, you must have your equation arranged in the form "(quadratic) = 0". Also, the "2a" in the denominator of the Formula is underneath everything above, not just the square root. And it's a "2a" under there, not just a plain "2". Make sure that you are careful not to drop the square root or the "plus/minus" in the middle of your calculations, or I can guarantee that you will forget to "put them back in" on your test, and you'll mess yourself up. Remember that " b^2 " means "the square of ALL of b , including its sign", so don't leave b^2 being negative, even if b is negative, because the square of a negative is a positive.

In other words, don't be sloppy and don't try to take shortcuts, because it will only hurt you in the long run. Trust me on this!

Here are some examples of how the Quadratic Formula works:

- **Solve $x^2 + 3x - 4 = 0$**

This quadratic happens to factor:

$$x^2 + 3x - 4 = (x + 4)(x - 1) = 0$$

...so I already know that the solutions are $x = -4$ and $x = 1$. How would my solution look in the Quadratic Formula? Using $a = 1$, $b = 3$, and $c = -4$, my solution looks like this:

$$\begin{aligned} x &= \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)} \\ &= \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{25}}{2} \\ &= \frac{-3 \pm 5}{2} = \frac{-3 - 5}{2}, \frac{-3 + 5}{2} \\ &= \frac{-8}{2}, \frac{2}{2} = -4, 1 \end{aligned}$$



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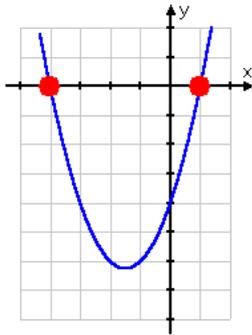
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Then, as expected, the solution is $x = -4$, $x = 1$.

Suppose you have $ax^2 + bx + c = y$, and you are told to plug zero in for y . The corresponding x -values are the x -intercepts of the graph. So solving $ax^2 + bx + c = 0$ for x means, among other things, that you are trying to find x -intercepts. Since there were two solutions for $x^2 + 3x - 4 = 0$, there must then be two x -intercepts on the graph. Graphing, we get the curve below:

As you can see, the x -intercepts (the red dots above) match the solutions, crossing the x -axis at $x = -4$ and $x = 1$. This shows the connection between graphing and solving: When you are solving "(quadratic) = 0", you are finding the x -intercepts of the graph. This can be useful if you have a graphing calculator, because you can use the Quadratic Formula (when necessary) to solve a quadratic, and then use your graphing calculator to make sure that the displayed x -intercepts have the same decimal values as do the solutions provided by the Quadratic Formula.



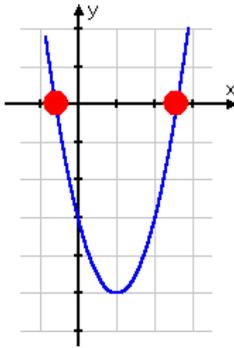
Note, however, that the calculator's display of the graph will probably have some pixel-related round-off error, so you'd be checking to see that the computed and graphed values were reasonably close; don't expect an exact match.

Solve $2x^2 - 4x - 3 = 0$. Round your answer to two decimal places, if necessary.

There are no factors of $(2)(-3) = -6$ that add up to -4 , so I know that this quadratic cannot be factored. I will apply the Quadratic Formula. In this case, $a = 2$, $b = -4$, and $c = -3$:

Then the answer is $x = -0.58$, $x = 2.58$, rounded to two decimal places.

Warning: The "solution" or "roots" or "zeroes" of a quadratic are usually required to be in the "exact" form of the answer. In the example above, the exact form is the one with the square roots of ten in it. You'll need to get a calculator approximation in order to graph the x -intercepts or to simplify the final answer in a word problem. But unless you have a good reason to think that the answer is supposed to be a rounded answer, always go with the exact



form. Compare the solutions of $2x^2 - 4x - 3 = 0$ with the x -intercepts of the graph:

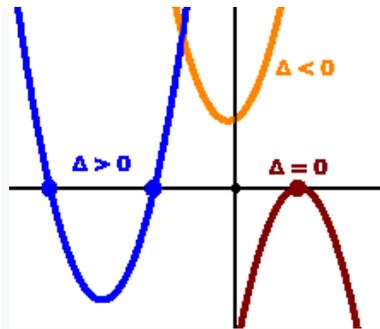
Just as in the previous example, the x -intercepts match the zeroes from the Quadratic Formula. This is always true. The "solutions" of an equation are also the x -intercepts of the corresponding graph.

Discriminant signs

In the quadratic formula, the expression underneath the square root sign is called the *discriminant* of the quadratic equation, and is often represented using an

upper case D or an upper case Greek delta:

$$\Delta = b^2 - 4ac$$



A quadratic equation with *real* coefficients can have either one or two distinct real roots, or two distinct complex roots. In this case the discriminant determines the number and nature of the roots. There are three cases:

- If the discriminant is positive, then there are two distinct roots

$$\frac{-b + \sqrt{\Delta}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{\Delta}}{2a}$$



both of which are real numbers. For quadratic equations with rational coefficients, if the discriminant is a square number, then the roots are rational—in other cases they may be quadratic irrationals.

- If the discriminant is zero, then there is exactly one real root

$$\frac{-b}{2a}$$

sometimes called a repeated or double root.

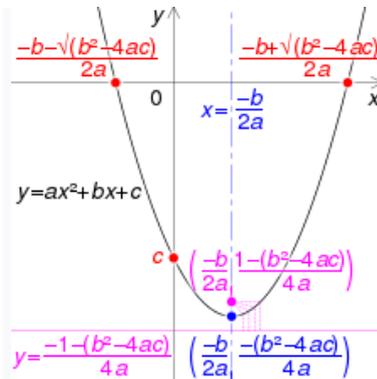
- If the discriminant is negative, then there are no real roots. Rather, there are two distinct (non-real) complex roots^[10]

$$\frac{-b+i\sqrt{-\Delta}}{2a} \quad \text{and} \quad \frac{-b-i\sqrt{-\Delta}}{2a}$$

which are complex conjugates of each other. In these expressions i is the imaginary unit.

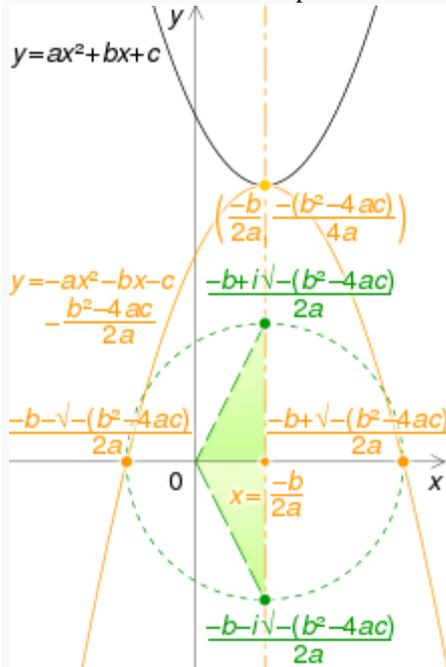
Thus the roots are distinct if and only if the discriminant is non-zero, and the roots are real if and only if the discriminant is non-negative.

Geometric interpretation



Graph of $y = ax^2 + bx + c$, where a and the discriminant $b^2 - 4ac$ are positive, with

- Roots and y -intercept in red
- Vertex and axis of symmetry in blue
- Focus and directrix in pink



Visualisation of the complex roots of $y = ax^2 + bx + c$: the parabola is rotated 180° about its vertex (orange). Its x -intercepts are rotated 90° around their midpoint, and the Cartesian plane is interpreted as the complex plane (green).^[11]

The function $f(x) = ax^2 + bx + c$ is the quadratic function.^[12] The graph of any quadratic function has the same general shape, which is called a parabola. The location and size of the parabola, and how it opens, depend on the values of a , b , and c . As shown in Figure 1, if $a > 0$, the parabola has a minimum point and opens upward. If $a < 0$, the parabola has a maximum point and opens downward. The extreme point of the parabola, whether minimum or maximum, corresponds to its vertex. The x -coordinate of the vertex will be located at $(-b/2a, -(b^2-4ac)/4a)$, and the y -coordinate of the vertex may be found by substituting this x -value into the function. The y -intercept is located at the point $(0, c)$.



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The solutions of the quadratic equation $ax^2 + bx + c = 0$ correspond to the roots of the function $f(x) = ax^2 + bx + c$, since they are the values of x for which $f(x) = 0$. As shown in Figure 2, if a , b , and c are real numbers and the domain of f is the set of real numbers, then the roots of f are exactly the x-coordinates of the points where the graph touches the x -axis. As shown in Figure 3, if the discriminant is positive, the graph touches the x -axis at two points; if it is zero, the graph touches at one point; and if it is negative, the graph does not touch the x -axis.

Examples and applications



The trajectory of the cliff jumper is parabolic because horizontal displacement is a linear function of time $x(t) = v_x t$, while vertical displacement is a quadratic function of time $y(t) = h_0 + v_y t + \frac{1}{2} g t^2$. As a result, the path follows a quadratic equation

The golden ratio is found as the positive solution of the quadratic equation $x^2 - x - 1 = 0$

The equations of the circle and the other conic sections—ellipses, parabolas, and hyperbolas—are quadratic equations in two variables.

Given the cosine or sine of an angle, finding the cosine or sine of the angle that is half as large involves solving a quadratic equation.

The process of simplifying expressions involving the square root of an expression involving the square root of another expression equates to finding the two solutions of a quadratic equation.



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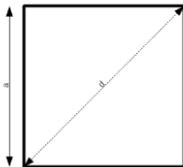
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Descartes' theorem states that for every four kissing (mutually tangent) circles, their radii satisfy a particular quadratic equation.

The equation given by Fuss' theorem, giving the relation among the radius of a bicentric quadrilateral's inscribed circle, the radius of its circumscribed circle, and the distance between the centers of those circles, can be expressed as a quadratic equation for which the distance between the two circles' centers in terms of their radii is one of the solutions. The other solution of the same equation in terms of the relevant radii gives the distance between the circumscribed circle's center and the center of the ex-circle of an ex-tangential quadrilateral.

Geometric Shapes - Areas

Square

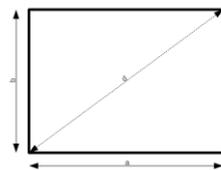


The area of a square can be calculated as $A = a^2$

Rectangle

The area of a rectangle can be calculated as

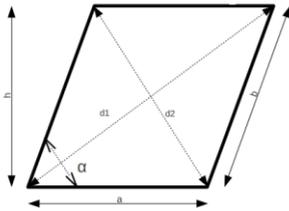
$$A = a b$$



Parallelogram

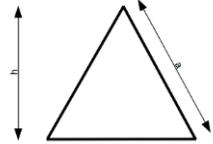
The area of a parallelogram can be calculated as

$$A = a h = a b \sin \alpha$$

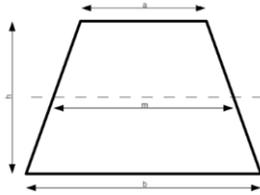


Triangle

The area of a triangle can be calculated as



$$A = a h / 2$$



Trapezoid

The area of a trapezoid can be calculated

as

$$A = 1/2 (a + b) h = m h$$

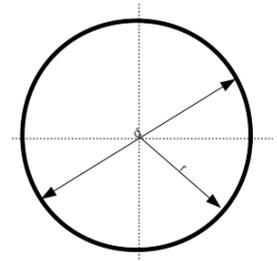
Circle

The area of a circle can be calculated as

$$A = \pi/4 d^2 = \pi r^2$$

$$C = 2 \pi r = \pi d$$

Where C = circumference



Sphere

Lateral surface area of a sphere can be expressed as

$$A = 4 \pi r^2$$



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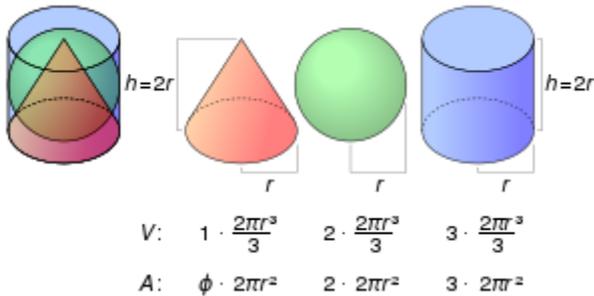
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Surface areas of common solids

Shape	Equation	Variables
<u>Cube</u>	$6s^2$	s = side length
<u>Cuboid</u>	$2(lw+lh+wh)$	l = length, w = width, h = height
<u>Triangular prism</u>	$bh+l(a+b+c)$	b = base length of triangle, h = height of triangle, l = distance between triangular bases, a, b, c = sides of triangle
All <u>prisms</u>	$2B+Ph$	B = the area of one base, P = the perimeter of one base, h = height
<u>Sphere</u>	$4\pi r^2$ or πd^2	r = radius of sphere, d = diameter





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Circle Definitions

The circle is the geometric place of all points from a plane situated at the same distance r from a point O , named the centre of the circle.

Circumference: The circumference of a circle is the distance around it.

Diameter: Any straight line that passes through the centre of the circle to two points on the perimeter ($2r$).

Radius: Any straight line that originates at the centre of a circle and ends at the perimeter (r).

Chord: A straight line whose ends are on the perimeter of a circle. A diameter is the longest possible chord.

Segment: A part of the circle separated from the rest of a circle by a chord.

Tangent: A tangent to a circle is a straight line which touches the circle at only one point (so it does not cross the circle - it just touches it).

Point of contact: Where a tangent touches a circle.

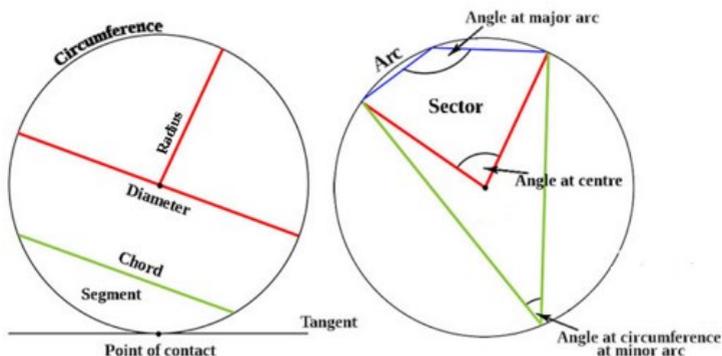
Arc: A part of the curve along the perimeter of a circle.

Angle on major arc: The larger of 2 angles when a circle is split into 2 uneven parts. Greater than 180 degrees.

Angle of centre: An angle at the centre of a triangle between two lines that intersect with the perimeter.

Angle at circumference on minor arc: The smaller of 2 angles when a circle is split into 2 uneven parts. Less than 180 degrees.

Sector: A portion of a circle resembling a 'slice of pizza'.





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In other words, area of sector = area of circle $\times A/360$ ($\pi r^2 \times A/360$)
arc length = circumference of circle $\times A/360$ ($2\pi r \times A/360$)

Operations on vectors

A vector is an object that has both a magnitude and a direction. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction. The direction of the vector is from its tail to its head.

Two vectors are the same if they have the same magnitude and direction. This means that if you take a vector and translate it to a new position (without rotating it), then the vector we obtain at the end of this translation is the same vector we had in the beginning.



10

Two examples of vectors are those that represent force and velocity. Both force and velocity are in a particular direction. The magnitude of the vector would indicate the strength of the force or the speed associated with the velocity.

We denote vectors using boldface as in \mathbf{aa} or \mathbf{bb} . Especially when writing by hand where one cannot easily write in boldface, people will sometimes denote vectors using arrows as in \vec{a} or \vec{b} , or they use other markings. We won't need to use arrows here. We denote the magnitude of the vector \mathbf{aa} by $\|\mathbf{a}\|$ or $\|\mathbf{a}\|$. When we want to refer to a number and stress that it is not a vector, we can call the number a scalar. We will denote scalars with italics, as in aa or bb .

You can explore the concept of the magnitude and direction of a vector using the below applet. Note that moving the vector around doesn't change the vector, as the position of the vector doesn't affect the magnitude or the direction. But if you stretch or turn the vector by moving just its head or its tail, the magnitude or direction will change.

There is one important exception to vectors having a direction. The zero vector, denoted by a boldface $\mathbf{00}$, is the vector of zero length. Since it has no length, it is not pointing in any particular direction. There is only one vector of zero length, so we can speak of **the** zero vector.



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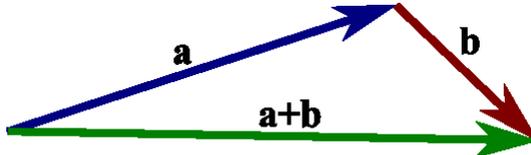
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Operations on vectors

We can define a number of operations on vectors geometrically without reference to any coordinate system. Here we define addition, subtraction, and multiplication by a scalar.

Addition of vectors

Given two vectors a and b , we form their sum $a+b$, as follows. We translate the vector b until its tail coincides with the head of a . (Recall such translation does not change a vector.) Then, the directed line segment from the tail of a to the head of b is the vector $a+b$.



The vector addition is the way forces and velocities combine. For example, if a car is travelling due north at 20 miles per hour and a child in the back seat behind the driver throws an object at 20 miles per hour toward his sibling who is sitting due east of him, then the velocity of the object (relative to the ground!) will be in a north-easterly direction. The velocity vectors form a right triangle, where the total velocity is the hypotenuse. Therefore, the total speed of the object (i.e., the magnitude of the velocity vector) is $\sqrt{20^2 + 20^2} = 20\sqrt{2}$ miles per hour relative to the ground.

Addition of vectors satisfies two important properties.

1. The commutative law, which states the order of addition doesn't matter
2. The associative law, which states that the sum of three vectors does not depend on which pair of vectors is added first

Vector subtraction

Before we define subtraction, we define the vector $-a$, which is the opposite of a . The vector $-a$ is the vector with the same magnitude as a but that is pointed in the opposite direction.



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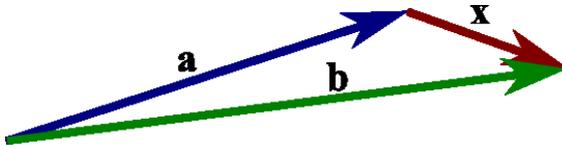


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We define subtraction as addition with the opposite of a vector:
 $b - a = b + (-a)$.

This is equivalent to turning vector a around in the applying the above rules for addition. Can you see how the vector x in the below figure is equal to $b - a$? Notice how this is the same as stating that $a + x = b$, just like with subtraction of scalar numbers.



Scalar multiplication

Given a vector a and a real number (scalar) λ , we can form the vector λa as follows. If λ is positive, then λa is the vector whose direction is the same as the direction of a and whose length is λ times the length of a . In this case, multiplication by λ simply stretches (if $\lambda > 1$) or compresses (if $0 < \lambda < 1$) the vector a .

If, on the other hand, λ is negative, then we have to take the opposite of a before stretching or compressing it. In other words, the vector λa points in the opposite direction of a , and the length of λa is $|\lambda|$ times the length of a . No matter the sign of λ , we observe that the magnitude of λa is $|\lambda|$ times the magnitude of a : $\|\lambda a\| = |\lambda| \|a\|$.

Theoretical background in physics

The applications present in this brochure, as well as those accessible on the Middle platform focus mainly on movements of bodies, more precisely on the



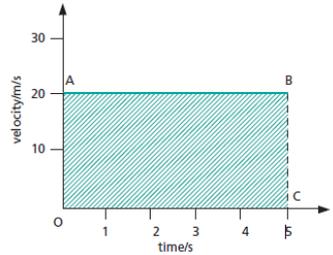
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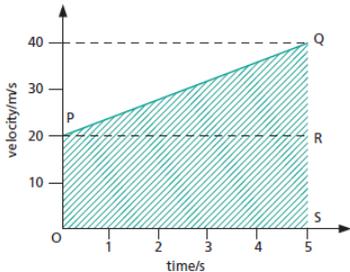
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rectilinear uniform movement of a body determined by its weight: vertical free fall, throwing an object obliquely or horizontally. Due to the fact that these may find applications in the launch of satellites, there is also a lesson presenting the necessary conditions for the launch, which also deals with knowledge of uniform circular movement. Each movement is influenced by its surroundings and the system of reference, as well as the combination of movements (through vectors), which is explained in a different lesson.



Velocity–time graphs

If the velocity of a body is plotted against time, the graph obtained is a **velocity–time graph**. It provides a way of solving motion problems. Tape charts are crude velocity–time graphs that show the velocity changing in jumps rather than smoothly, as occurs in practice. A motion sensor gives a smoother plot. The area under a velocity–time graph measures the distance travelled. In Figure 1 AB is the velocity–time graph for a body moving with a **uniform velocity** of 20 m/s. Since distance = average velocity \times time, after 5 s it will



have moved $20 \text{ m/s} \times 5 \text{ s} = 100 \text{ m}$. This is the shaded area under the graph, i.e. rectangle OABC.

In Figure 2 PQ is the velocity–time graph for a body moving with **uniform acceleration**. At the start of the timing the velocity is 20 m/s but it increases steadily to 40 m/s after 5 s. If the distance covered equals the area under PQ, i.e. the shaded area OPQS, then

distance = area of rectangle OPRS + area of triangle PQR = $OP \times OS + 1/2 \times PR \times QR$ (area of a triangle = $1/2 \text{base} \times \text{height}$) = $20 \text{ m/s} \times 5 \text{ s} + 1/2 \times 5 \text{ s} \times 20 \text{ m/s} = 100 \text{ m} + 50 \text{ m} = 150 \text{ m}$
time/s

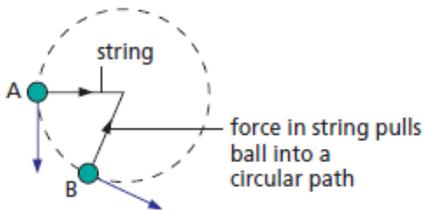


In Figure 1, the slope of AB is zero, as is the acceleration. In Figure 2, the slope of PQ is $QR/PR = 20/5 = 4$: the acceleration is 4 m/s^2 .

Circular motion

There are many examples of bodies moving in circular paths – rides at a funfair, clothes being spun dry in a washing machine, the planets going round the Sun and the Moon circling the Earth. When a car turns a corner it may follow an arc of a circle. ‘Throwing the hammer’ is a sport practised at highland games in Scotland, in which the hammer is whirled round and round before it is released.

Centripetal force



In the figure, a ball attached to a string is being whirled round in a horizontal circle. Its direction of motion is constantly changing. At A it is along the tangent at A; shortly afterwards, at B, it is along the tangent at B; and so on. Velocity has both size and direction; speed has only size. Velocity is speed in a stated

direction and if the direction of a moving body changes, even if its speed does not, then its velocity has changed. A change of velocity is an acceleration, and so during its whirling motion the ball is accelerating. It follows from Newton’s first law of motion that if we consider a body moving in a circle to be accelerating then there must be a force acting on it to cause the acceleration. In the case of the whirling ball it is reasonable to say the force is provided by the string pulling inwards on the ball. Like the acceleration, the force acts towards the centre of the circle and keeps the body at a fixed distance from the centre.

A larger force is needed if

- (i) the speed v of the ball is increased,
- (ii) the radius r of the circle is decreased,
- (iii) the mass m of the ball is increased.

The rate of change of direction, and so the acceleration a , is increased by (i) and (ii). It can be shown that

$$a = v^2/r \text{ and so, from } F = ma,$$



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we can write $F = mv^2/r$

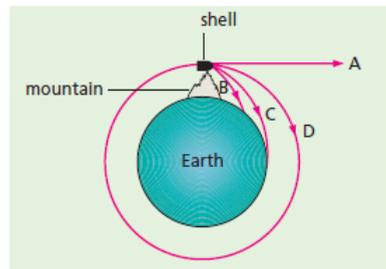
This force, which acts **towards the centre** and keeps a body moving in a circular path, is called the **centripetal force** (centre-seeking force). Should the force be greater than the string can bear, the string breaks and the ball flies off i.e. in the direction of travel when the string broke (as Newton's first law of motion predicts). It is not thrown outwards.

Whenever a body moves in a circle (or circular arc) there must be a centripetal force acting on it. In throwing the hammer it is the pull of the athlete's arms acting on the hammer towards the centre of the whirling path. When a car rounds a bend a frictional force is exerted inwards by the road on the car's tyres.

Satellites

For a **satellite of mass m** orbiting the Earth at radius r with orbital speed v , the centripetal force, $F = mv^2/r$, is provided by gravity. To put an artificial satellite in orbit at a certain height above the Earth it must enter the orbit at the correct speed. If it does not, the force of gravity, which decreases as height above the Earth increases, will not be equal to the centripetal force needed for the orbit.

This can be seen by imagining a shell fired horizontally from the top of a very high mountain. If gravity did not pull it towards the centre of the Earth it would continue to travel horizontally, taking path A. In practice it might take path B. A second shell fired faster might take path C and travel further. If a third shell is fired even faster, it might never catch up with the rate at which the Earth's



surface is falling away. It would remain at the same height above the Earth (path D) and return to the mountain top, behaving like a satellite. The orbital period T (the time for one orbit) of a satellite = distance/velocity. So for a circular orbit $T = 2\pi r/v$

Satellites in high orbits have longer periods than those in low orbits. The Moon is kept in a circular orbit round the Earth by the force of gravity between it and the Earth. It has an orbital period of 27 days.

a) Communication satellites

These circle the Earth in orbits above the equator. **Geostationary satellites** have an orbit high above the equator (36 000 km); they travel with the same



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speed as the Earth rotates, so appear to be stationary at a particular point above the Earth's surface – their orbital period is 24 hours. They are used for transmitting television, intercontinental telephone and data signals. Geostationary satellites need to be well separated so that they do not interfere with each other; there is room for about 400. Mobile phone networks use many satellites in much lower equatorial orbits; they are slowed by the Earth's atmosphere and their orbit has to be regularly adjusted by firing a rocket engine. Eventually they run out of fuel and burn up in the atmosphere as they fall to Earth.

b) Monitoring satellites

These circle the Earth rapidly in low polar orbits, i.e. passing over both poles; at a height of 850 km the orbital period is only 100 minutes. The Earth rotates below them so they scan the whole surface at short range in a 24-hour period and can be used to map or monitor regions of the Earth's surface which may be inaccessible by other means. They are widely used in weather forecasting to continuously transmit infrared pictures of cloud patterns down to Earth, which are picked up in turn by receiving stations around the world.

Questions

A satellite close to the Earth (at a height of about 200 km) has an orbital speed of 8 km/s. Taking the radius of the orbit as approximately equal to the Earth's radius of 6400 km, calculate the time it takes to make one orbit.

Throwing an object obliquely

A projectile is any object that is cast, fired, flung, heaved, hurled, pitched, tossed, or thrown. (This is an informal definition.) The path of a projectile is called its trajectory. Some examples of projectiles include...

- a baseball that has been pitched, batted, or thrown
- a bullet the instant it exits the barrel of a gun or rifle
- a bus driven off an uncompleted bridge
- a moving airplane in the air with its engines and wings disabled
- a runner in mid stride (since they momentarily lose contact with the ground)
- the space shuttle or any other spacecraft after main engine cut off (MECO)



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The force of primary importance acting on a projectile is gravity. This is not to say that other forces do not exist, just that their effect is minimal in comparison. A tossed helium-filled balloon is not normally considered a projectile as the drag and buoyant forces on it are as significant as the weight. Helium-filled balloons can't be thrown long distances and don't normally fall. In contrast, a crashing airplane would be considered a projectile. Even though the drag and buoyant forces acting on it are much greater in absolute terms than they are on the balloon, gravity is what really drives a crashing airplane. The normal amounts of drag and buoyancy just aren't large enough to save the passengers on a doomed flight from an unfortunate end. A projectile is any object with an initial non-zero, horizontal velocity whose acceleration is due to gravity alone.

An essential characteristic of a projectile is that its future has already been preordained. Batters may apply "body English" after hitting a long ball, but they do so strictly for psychological reasons. No amount of leaning to one side will make a foul ball turn fair. Of course, the pilot of a disabled airplane may regain control before crashing and avert disaster, but then the airplane wouldn't be a projectile anymore. An object ceases to be a projectile once any real effect is made to change its trajectory. The trajectory of a projectile is thus entirely determined the moment it satisfies the definition of a projectile.

The only relevant quantities that might vary from projectile to projectile then are initial velocity and initial position

This is where we run into some linguistic complications. Airplanes, guided missiles, and rocket-propelled spacecraft are sometimes also said to follow a trajectory. Since these devices are acted upon by the lift of wings and the thrust of engines in addition to the force of gravity, they are not really projectiles. To get around this dilemma, it is common to use the term **ballistic trajectory** when dealing with projectiles. The word ballistic has its origins in the Greek word βαλλω (*vallo*), to throw, and surfaces repeatedly in the technical jargon of weaponry from ancient to modern times. For example...

- The **ballista**, which looks something like a giant crossbow, was a siege engine used in medieval times to hurl large stones, flaming bundles, infected animal carcasses, and severed human heads into fortifications. Before the invention of gunpowder, ballista's (and catapults and trebuchets) were the weapons of choice for conquerors.
- An **intercontinental ballistic missile** is a device for delivering nuclear warheads over long distances. At the start of its journey an ICBM is guided by a rocket engine and stabilizer fins, but soon thereafter it enters the



phase of its journey where it is effectively in free fall, traveling fast enough to keep it above the earth's atmosphere for a while but not fast enough to enter orbit permanently. The adjective "intercontinental" refers to the long range capabilities, while the largely free fall journey it takes makes it "ballistic". ICBMs are the ultimate killing machines, but they have never been used in combat to date.

The wide geographic range as well as the wide historic range of these things we call projectiles raises some problems for the typical student of Physics. When a projectile is sent on a very long journey, as is the case with ICBMs, the magnitude and direction of the acceleration due to gravity changes. Gravity isn't constant to begin with, but the variation is not noticeable over everyday ranges in altitude. From the deepest mines in South Africa to the highest altitudes traversed by commercial airplanes, the *magnitude* of the acceleration due to gravity is always effectively $9.8 \text{ m/s}^2 \pm 0.05 \text{ m/s}^2$. Similarly, unless you routinely travel medium to long distances, you aren't likely to experience much of a change in the *direction* of gravity either. To experience a 1° shift in "down" would require traveling $\frac{1}{360}$ of the circumference of the Earth — roughly 110 km (70 mi) or the length of a typical morning commute to work in Southern California. Thus for projectiles that won't rise higher than an airplane nor travel farther than the diameter of L.A., gravity is effectively constant. This covers the first five of the examples described at the beginning of this section (baseballs, bullets, buses in action-adventure movies, distressed airplanes, and joggers) but not the sixth (the space shuttle after MECO).

To distinguish such simple projectiles from those where variations in gravity and the curvature of the Earth are significant, I propose using the term simple projectile. For the remaining problems, the term general projectile seems appropriate since a general solution in Mathematics is one that also includes the special cases, but I'm less adamant about this term.

Consider an effectively spherical earth with a single tall mountain sticking out of it like a giant tumor. Now imagine using this location as a place to launch projectiles horizontally with varying initial velocities. What effect would velocity have on range? Well, obviously fast projectiles will travel farther than slow ones. A basic concept associated with speed is that "faster means farther", but the relationship is only approximately linear on a spherical earth. For a while, doubling speed would mean doubling distance, but eventually the curvature of the Earth would start to mess things up. At some speed our hypothetical projectile would make it a quarter of the way around the Earth



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and then half way around and then eventually all the way around. At this point our general projectile ceases to be an object with a launch point and a landing point and it starts being a **satellite**, permanently circling the Earth, perpetually changing direction and thus accelerating under the influence of gravity, but never landing anywhere. Technically, such an object would still be a general projectile, since gravity is the primary source of its acceleration, but somehow this doesn't seem right. Objects traveling through what we call "outer space" hardly seem like projectiles any more. They seem like they reside more in the realm of **celestial mechanics** than **terrestrial mechanics**. Such distinctions are arbitrary, however, as there is only one mechanics. The laws of Physics are assumed universal until it can be demonstrated otherwise. The unification of physical law is a theme that surfaces from time to time in Physics.

A projectile and a satellite are both governed by the same physical principles even though they have different names. A simple projectile is made mathematically simple by an idealization. By assuming a constant value for the acceleration due to gravity, we make the problem easier to solve and (in many cases) do not really lose all that much in the way of accuracy. Every projectile problem is essentially two one-dimensional motion problems...

The kinematic equations for a simple projectile are those of an object traveling with constant horizontal velocity and constant vertical acceleration.

The equations of motion for a simple projectile

equation	horizontal	vertical
acceleration	$a_x = 0$	$a_y = -g$
velocity-time	$v_x = v_0x$	$v_y = v_0y - gt$
displacement-time	$x = x_0 + v_0xt$	$y = y_0 + v_0yt - \frac{1}{2}gt^2$
velocity-displacement		$v_y^2 = v_0y^2 - 2g(y - y_0)$



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The trajectory of a simple projectile is a parabola.
Use the horizontal direction to determine the range as a function of time...

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = 0 + (v \cos \theta) t + 0$$

$$x_{final} = (v \cos \theta) t_{final}$$



Use the vertical direction to determine the time in the air...

$$y = y_0 + v_{0y}t + a_y t^2 / 2$$

$$y = y_0 + (v \sin \theta)t - g t^2 / 2$$

$$0 = 0 + (v \sin \theta)t_{final} - g t_{final}^2 / 2$$

$$t_{final} = \frac{2(v \sin \theta)}{g}$$

Combine these two equations...

$$x_{final} = (v \cos \theta) \frac{2(v \sin \theta)}{g}$$

$$x_{final} = \frac{v^2 \sin 2\theta}{g} \quad x_{max} = 2x_{final}$$

Questions

1 A stone falls from rest from the top of a high tower. Ignore air resistance and take $g = 10 \text{ m/s}^2$. **a** What is its velocity after

(i) 1 s, **(ii)** 2 s, **(iii)** 3 s, **(iv)** 5 s?

b How far has it fallen after

(i) 1 s, **(ii)** 2 s, **(iii)** 3 s, **(iv)** 5 s?

2 An object falls from a hovering helicopter and hits the ground at a speed of 30 m/s. How long does it take the object to reach the ground and how far does it fall? Sketch a velocity–time graph for the object (ignore air resistance).

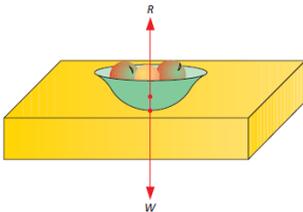


Forces and resultants

A **vector** quantity is one such as force which is described completely only if both its size (magnitude) and direction are stated. It is not enough to say, for example, a force of 10 N, but rather a force of 10 N acting vertically downwards. A vector can be represented by a straight line whose length represents the magnitude of the quantity and whose direction gives its line of action. An arrow on the line shows which way along the line it acts.

A **scalar** quantity has magnitude only. Mass is a scalar and is completely described when its value is known. Scalars are added by ordinary arithmetic; vectors are added geometrically, taking account of their directions as well as their magnitudes.

Force has both magnitude (size) and direction. It is represented in diagrams by a straight line with an arrow to show its direction of action. Usually more than



one force acts on an object. As a simple example, an object resting on a table is pulled downwards by its weight W and pushed upwards by a force R due to the table supporting it. Since the object is at rest, the forces must balance, i.e. $R = W$.

Parallelogram law

in size and direction by the from the point, their resultant direction by the diagonal of the point.

Examples of addition of
1 Two people carrying a

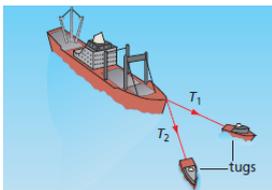
If two forces acting at a point are represented



sides of a parallelogram drawn is represented in size and the parallelogram drawn from

forces

heavy bucket. The weight of the bucket is balanced by the



force F , the resultant of F_1 and F_2

2 Two tugs pulling a ship. The resultant of T_1 and T_2 is forwards in direction and so the ship moves forwards (as long as the resultant is greater than the resistance to motion of the sea and the wind).

Questions

1 Jo, Daniel and Helen are pulling a metal ring. Jo pulls with a force of 100 N in one direction and Daniel with a force of 140 N in the opposite direction. If the ring does not move, what force does Helen exert if she pulls in the same direction as Jo?



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- 2 A boy drags a suitcase along the ground with a force of 100 N. If the frictional force opposing the motion of the suitcase is 50 N, what is the resultant forward force on the suitcase?
- 3 A picture is supported by two vertical strings; if the weight of the picture is 50 N what is the force exerted by each string?
- 4 Using a scale of 1 cm to represent 10 N, find the size and direction of the resultant of forces of 30 N and 40 N acting at right angles to each other.
- 5 Find the size of the resultant of two forces of 5 N and 12 N acting
 - a in opposite directions to each other,
 - b at 90° to each other.



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1. Projectile Motion with an Initial Horizontal Velocity

Field of application: Physics

Required knowledge: Calculations with real numbers, basic Mathematical operations, second grade equations, uniform rectilinear motion

Project: understanding the influence of gravity over a body's movement; applying the knowledge to the real world, with an experiment

Moodle:

<http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=3#section-1>

Authors: Aspasia Vogiatzi, Georgios Kottas, students from Io Geniko Lykeio, Aigiou, Greece

Coordinator: Nikolaos Diamantopoulos

The assignment: of this lesson is to study the movement of a ball with initial horizontal velocity at a certain height

Resources: Information sheet – Source: https://www.youtube.com/watch?v=iNEYo_UNFYQ.

Generalization: Research can be extended to any body being thrown at a certain height horizontally, under the influence of gravity.





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2. Speed and Distance

Field of application: Physics

Required knowledge: calculating areas, graphing first grade functions and interpreting their intersections with the axes (OX, OY)

Project: familiarizes students with the link between velocity, time and their graphical representation. The area under a graph often gives useful information.

In this activity you will explore one example of this, the area under a speed–time graph.

Moodle:

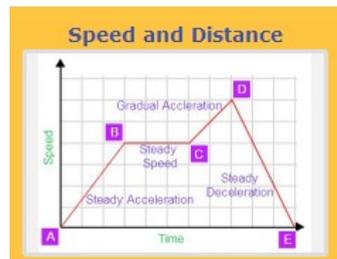
<http://srv-11yk-aigiou.ach.sch.gr/moodle/course/view.php?id=3#section-8>

Authors: Evangelia Karatza, Dimitra Skoura, students from Io Geniko Lykeio, Aigiou, Greece

Coordinator: Nikolaos Diamantopoulos, Spyridon Potamitis

The assignment: plotting the velocity of a body against time, in order to realise that the area of the figure obtained gives the value of the distance travelled

Generalization: it may be shown that work, electric charge in a conductor etc can be calculated with the help of the area.





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3. MARCO the DREAMer

Field of application: Physics

Required knowledge: solving second grade equations, square roots, basic calculations

Project: Marco survives due to his knowledge of Physics (Mechanics and Thermodynamics) and Mathematics

<http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=3#section-6>

Authors: Azgur Ariana, Grigoraş Flavia, Iorga Patricia, Paşca Bianca, Petcu Miruna, Colegiul National "C.D. Loga" Timisoara

Coordinator: prof. Stoia Simona, Samfirescu Isabela, România

The problem: I'm stuck on a rock...how do I come down? I found myself at a big distance from the ground, so I had to use the rope I had in my backpack. To make sure that it was going to hold me, I used the 'equilibrium of forces'. I decided to tie my rope, which was 50m long, around a stone and throw it down, to figure out the distance to the plateau. I realized that my rope was 3m too long, so using Maths I calculated the distance to the ground: 47m. To find out the gravitational acceleration, I threw a small rock and started the timer I had in my backpack. It took 3,19s for the rock to reach the ground, so all I had to do was use the formulas.



Resources: The document

Generalization: It may be considered a technique of survival ... but not in extreme conditions (i.e. do try this, only at home :))



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Beneficiary: Colégio Nacional "Constantin Diaconovici Loga", Timisoara

4. Angry Birds Projectile Motion

Field of application: Physics

Required knowledge: Calculations with real numbers, solving second grade equations

Project: Angry Birds is a prime example of projectile motion in everyday situations. Millions of people have played this game and without even knowing it, calculated projectile motion.

Moodle:

<http://srv-11yk-aigiou.ach.sch.gr/moodle/course/view.php?id=3#section-4>

Authors: Georgios Kottas, Andreas Bodiotis Io Geniko Lykeio Aigiou, Grece.

Coordinator: Nikolaos Diamantopoulos, Ilias Spanos,

The problem: Use <http://screencast-o-matic.com/> site to capture a video of a yellow angry birds 1st level shot. Make sure before shooting to minimize the angry birds screen so that the background doesn't move. Measure the horizontal velocity by hand. Measure the vertical velocity by hand.

Follow a similar procedure as above for the calculation of vertical velocity, according to Figure 2 below.

What do you observe about the time it takes for the angry bird to travel in the y direction?

The sling's angle changes, having an effect on the bird's movement, which can be studied .

Resources:

<http://physlets.org/tracker/>

Generalization: The movement of bodies under the influence of gravity when their initial velocity is at an angle from the horizontal direction can be studied.





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Applications of Maths – DREAM"
Beneficiary: Colegiul Național "Constantin Dăscălescu" Lugoj, Timișoara

5. Movement of artificial satellites around Earth

Field of application: Physics

Required knowledge: notions of Euclidian geometry and elementary Maths

Project: Explaining the physical variables upon which depends launching and stabilising an artificial satellite orbiting around Earth . => Kepler's Laws

Moodle:

<http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=3#section-3>

Authors: Portugal Team, Agrupamento de Escolas Soares Basto, Portugal

Coordinator: Prof. Paula Cristina Sousa Pereira Ornelas

The problem: Calculate the periods of cyclical movements of the satellites of the previous question. Check your calculations.



Help: Introduce a digital stopwatch in the display window to be able to calculate with reasonable accuracy the traveled time. With the pause button in the control window, you can temporarily stop the simulation in order to measure the time. In a chart window you can display graphs of each coordinate of any satellite, over time. From these graphs, you

can easily calculate the periods of their movements.

Resources:

http://srv-1lyk-aigiou.ach.sch.gr/moodle/pluginfile.php/82/mod_resource/content/1/solution_modellus

Generalization: This type of movement can be studied in the planets in our Solar System.



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Beneficiary: Colegiul Național "Constantin Diaconovici Loga", Timișoara

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